Introduction to Differential Geometry

Felix Zhou [1]

January 5, 2024

[1]Based on "Differential Geometry: Connections, Curvature, and Characteristic Classes" by Loring Tu

Contents

Chapter 1

Curvature & Vector Fields

By a manifold, we exclusively refer to a smooth manifold. The theory of smooth manifolds focuses on the *differential topology* of a manifold, where the main properties are preserved under stretching. In *differential geometry*, we endow a manifold with a *Riemannian metric*, which gives a way to measure length. This does not make sense in the setting of differential topology.

1.1 Riemannian Manifolds

A *Riemannian metric* is essentially a smoothly varying inner product on the tangent space at each point of a manifold. In this section, we recall some generalities about an inner product on a vector space and by means of a partition of unity argument, prove the existence of a Riemannian metric on any manifold.